Minimal Centroaffine Immersions of Codimension Two

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Abstract

Minimal equi-centroaffine immersions of codimension two are characterized as solutions of a certain variational problem. We determine the moduli space of such immersions of $\mathbb{R}^2$ into $\mathbb{R}^4$ whose induced connection and affine fundamental form coincide with the ones of the Clifford torus.

1 Introduction

In equi-centroaffine differential geometry, the theory of hypersurfaces has a long history. However, relatively little has been achieved in the study of equi-centroaffine immersions with higher codimensions. It is the purpose of this paper to study such immersions with codimension two.

For a centroaffine immersion into the affine space, the position vector yields its *first canonical* normal vector field. A standard method of choosing a *second* one was proposed in 1950 by Lopšic (see Walter [7]). Recently, reorganizing geometry of equi-centroaffine immersions of codimension two, Nomizu and Sasaki [4] proposed another fruitful choice. Adopting the latter one, we take the prenormalized Blaschke normal field as the second canonical normal vector field. Fixing two normal vector fields, we can define the induced volume form and consider a variational problem of the volume. Then we say that an equi-centroaffine immersion of codimension two is *minimal* if the volume is extremal under any variation having no part in the direction of the position vector infinitesimally. We prove that an equi-centroaffine

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