On the Classical d-Orthogonal Polynomials Defined by Certain Generating Functions , I

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Abstract

The purpose of this work is to present some results on the d-orthogonal polynomials defined by generating functions of certain forms to be specified below. The resulting polynomials are natural extensions of some classical orthogonal polynomials. The first part of this study is motived by the recent work of Von Bachhaus [21] who showed that, among the orthogonal polynomials, only the Hermite and the Gegenbauer polynomials are defined by the generating function $G[2xt - t^2]$. Here we generalize this result in the context of d-orthogonality, by considering the polynomials generated by $G[(d+1)xt - t^{d+1}]$, where d is a positive integer. We obtain that the resulting polynomials are d-symmetric (Definition 1.2) and "classical" in the Hahn's sense. We provide some examples to illustrate the results obtained and show that they involve certain known polynomials. Finally, we conclude by giving some properties of the zeros of these polynomials as well as a $(d+1)$-order differential equation satisfied by each polynomial. In forthcoming paper [2] we will consider the polynomials generated by $e^t\Psi(xt)$.

1 Introduction and preliminary results

Let $G[z]$ be analytic at $z = 0$ and has the expansion

$$G[z] = \sum_{n=0}^{\infty} a_n z^n , \quad a_n \neq 0 , \; n \geq 0. \tag{1.1}$$

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