Derivable affine planes and translation planes

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Abstract

It was proved by Johnson that every derivable affine plane admits a natural embedding into a 3-dimensional projective space over some skewfield. We show that under this embedding the lines of the affine plane which do not belong to the derivation set correspond to spreads of the projective space. Furthermore, we investigate the spreads associated with derivable translation planes or derivable dual translation planes more closely. Finally, we study derivable affine planes admitting a so-called affine Hughes group.

1 Derivable affine planes and associated spreads

Let $\mathcal{A} = (A, \mathcal{G})$ be an affine plane and denote the projective extension of $\mathcal{A}$ by $\bar{\mathcal{A}} = (\bar{A}, \bar{\mathcal{G}})$, where $\bar{\mathcal{G}} = \mathcal{G} \cup \{L_\infty\}$. For a subset $D \subset L_\infty$ we put $\mathcal{G}(D) = \{L \in \mathcal{G} | L \wedge L_\infty \in D\}$ and we denote by $\mathcal{B}(D)$ the set of the Baer subplanes of $\mathcal{A}$ which intersect L_∞ in D. A subset $D \subset L_\infty$ is called a derivation set of $\mathcal{A}$ if for any two distinct points $p, q \in A$ with $p \vee q \in \mathcal{G}(D)$ there exists a Baer subplane $\mathcal{A}_{p,q,D}$ of $\mathcal{A}$ which contains p, q and D. By [7], Lemma 2.4 the Baer subplane $\mathcal{A}_{p,q,D}$ is uniquely determined by p, q and D, under the assumption that D is a derivation set. If D is a derivation set of $\mathcal{A}$ we can form a new affine plane $\mathcal{A}'$ with the same set of points by replacing the lines in $\mathcal{G}(D)$ with the Baer subplanes in $\mathcal{B}(D)$. An affine plane $\mathcal{A}$ admitting a derivation set is called derivable and $\mathcal{A}'$ is called the derived plane of $\mathcal{A}$ with respect to D. The plane $\mathcal{A}'$ can be derived in such a way that $\mathcal{A}''$ is isomorphic to $\mathcal{A}$.

Extending results of Cofman [3], Johnson [8] has shown that every derivable affine plane admits a natural embedding into a 3-dimensional projective space. Johnson's theorem is most easily formulated as follows.

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