m-convex properties in locally convex algebras

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Abstract

We consider m-convex properties and compare different radii in locally convex algebras.

Introduction

We say that the continuity of the inverse map $x \mapsto x^{-1}$ and the equicontinuity of the power maps $(x \mapsto x^n)_n$, are m-convex properties. They are essential in m-convex algebras. It is also known that several radii ρ, β and R_1 to R_7 are equal in such complete algebras ([5]). We prove the same in a more general situation and under the weakest completion notion (pseudo-completeness).

In section 2, we show, in particular, that in a unitary pseudo-complete locally convex algebra with continuous inverse, the equality of ρ, β and R_1 to R_7 is still true (Proposition 2.2). We also obtain $\rho = \beta$ in any unitary pseudo-complete locally convex algebra for which the power maps $(x \mapsto x^n)_n$ are equicontinuous at zero. In section 3, we examine links between the two m-convex properties above. In the general context of unitary locally convex algebra's, the equicontinuity of power maps implies the continuity of the inverse map under the additional condition that the map $x \mapsto R_7(x)$ be upper semi-continuous (Proposition 3.1). It appears that in a unitary pseudo-complete locally convex algebra (A, τ) which is a Q-algebra such that the power maps are equicontinuous at zero, the continuity of the inverse is equivalent to the boundedness of every element of (A, τ) (Proposition 3.2). In section 4, we generalize a result of A. Arosio ([3]) on B_0-algebras. We obtain that the power maps are equicontinuous at zero in any Baire locally convex algebra (A, τ) with a