Asymptotic properties of Abelian integrals arising in quadratic systems*

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Abstract

We consider quadratic perturbations of the vector field $(-y+ax^2+by^2)\partial_x + x(1+cy)\partial_y$ and study its limit cycles via Abelian integrals. The asymptotic analysis suggests that such systems have no more than 4 limit cycles.

1 Introduction

The 16-th Hilbert problem is to find a bound $N(n)$ for the number of limit cycles of planar vector fields of degree n. Even for quadratic systems the answer is unknown. There are examples [2], [9] of quadratic systems with 4 limit cycles. In the present paper the author examines the possibility of finding quadratic systems with >4 limit cycles in one specific situation.

We consider the vector field

$$\dot{x} = -y + ax^2 + by^2, \ \dot{y} = x(1+cy), \ c \leq 0 \tag{1}$$

which is time–reversible, (invariant under $(x, y, t) \to (-x, y - t)$), and has two centers: $x = y = 0$ and $x = 0, y = 1/b$, (see below). One can check that the center $(0, 0)$ has cyclicity 2 for $3a + 5b \neq c$ and 3 for $3a + 5b = c$, (see Section 5 below). The other center also has cyclicity 2 or 3. It seems that the configuration with 3 limit cycles around one focus and 2 cycles around the other focus for a perturbation of (1) is possible.

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