Existence of integrals for finite dimensional quasi-Hopf algebras

Florin Panaite Freddy Van Oystaeyen

1 Introduction

If A is a finite dimensional Hopf algebra and $\int \subseteq A$ is the space of integrals in A, it is well known that $dim(\int) = 1$. The proof given in [12] actually shows the existence and uniqueness of integrals in A^* and it relies on the structure of Hopf modules over A, namely one has to prove that A^* is a right A-Hopf module and then the result follows from the fundamental theorem for Hopf modules (see [12] for details).

It is very natural to ask if the result remains true if A is not a Hopf algebra, but a quasi-Hopf algebra (this question arose in [9], where the following version of Maschke's theorem for quasi-Hopf algebras was proved: A is semisimple if and only if $\varepsilon(\int) \neq 0$). The answer is positive for some particular quasi-Hopf algebras, for instance for Dijkgraaf-Pasquier-Roche's quasi-Hopf algebras $D^\omega(G)$ (where G is a finite group and ω is a normalized 3-cocycle on G) and for their generalizations $D^\omega(H)$ introduced in [1] (where H is a finite dimensional cocommutative Hopf algebra and $\omega : H \otimes H \otimes H \to k$ is a normalized 3-cocycle in Sweedler's cohomology). But if one tries to generalize the proof given in [12] to quasi-Hopf algebras some problems occur, for example it is not clear which could be the appropriate definition for a Hopf module over a quasi-Hopf algebra.

The existence and uniqueness of integrals for finite dimensional Hopf algebras have been reproved in [11], [8] by avoiding the use of Hopf modules. In this note we shall prove the *existence* of integrals for finite dimensional quasi-Hopf algebras, by generalizing the short and direct proof given by A. Van Daele in [11] for the Hopf algebra case. It seems that the method in [11] does not yield a proof for the *uniqueness* property.

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