Contribution to the modelling of the hump effect by the study of an equation of Hamilton-Jacobi type

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Abstract

This paper is a contribution to the mathematical modelling of the hump effect. We present a mathematical study (existence, homogenization) of an Hamilton-Jacobi problem which represents the propagation of a front flame in a striated media.

1 Introduction

The physical problem consists in an anomaly of overvelocity observed in the combustion room of propellers during the combustion of some solid propellants blocks. This anomaly, called 'Hump effect', attains its maximum in the middle of the burning block. The reduced mathematical model of this phenomenon (hump effect) is the following Hamilton-Jacobi problem:

$$P_\xi \quad \begin{cases} \dfrac{\partial \xi}{\partial t} + R_0(\xi, s_2)\sqrt{1+\left(\dfrac{\partial \xi}{\partial s_2}\right)^2} = 0 \quad \forall t > 0\ ,\ s_2 \in \mathbb{R} \\ \\ \xi(s_2, 0) = \xi_0(s_2) \qquad\qquad\qquad\qquad s_2 \in \mathbb{R} \end{cases}$$

where the unknown $s_1 = \xi(s_2, t)$ is the position of the flame front. We show in this paper that the anomaly results from the heterogeneity of the propellant blocks.

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