Reduction of Hopf bifurcation problems with symmetries

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Abstract

We consider a family of Γ -equivariant differential equations and look for Hopf bifurcation. We reformulate this problem in the usual way as an operator equation and perform a Liapunov-Schmidt reduction determined by the "semisimple" part of its linearization. In a second reduction step we construct a bifurcation equation which furtunately can be formulated directly by means of the original problem.

1 Introduction

We consider a family of autonomous differential equations

$$\dot{x} = f(x, \lambda) \tag{1}$$

where $x \in \mathbf{R}^n, \lambda \in \mathbf{R}^m, f : \mathbf{R}^n \times \mathbf{R}^m \to \mathbf{R}^n$ is of class C^∞ and $f(0, \lambda) = 0$, for all λ. Furthermore let the system (1) be Γ-equivariant, that is: there exists a compact group $\Gamma \subset O(n)$, such that

$$f(\gamma x, \lambda) = \gamma f(x, \lambda)\,,\ \forall \gamma \in \Gamma \text{ and } \forall (x, \lambda) \in \mathbf{R}^n \times \mathbf{R}^m.$$

We want to study Hopf bifurcation in the given family under the following condition: $A := D_1 f(0, 0)$ has eigenvalues μi for some integers $\mu \in \mathrm{Z}$ while 0 is not an eigenvalue

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