

# Pseudo-Riemannian metrics on tangent bundle and harmonic problems \*

Oniciuc C.

## Abstract

The problems studied in this paper are related to the harmonicity of the canonical projection  $\pi : TM \rightarrow M$ , where  $(M, g)$  is a Riemannian space and  $TM$  is its tangent bundle, and to the harmonicity of the vector fields  $\xi \in \chi(M)$  thought of as maps from  $M$  to  $TM$ . We have considered on  $TM$  the pseudo-Riemannian metrics  $G, g^c$  of lift-complete type defined by means of an arbitrary nonlinear connection on  $TM$ . We have also studied the harmonicity of a tensor field  $J$  of type  $(1, 1)$  on  $M$ , where  $J$  is thought of as a map from  $TM$  into itself.

## Introduction

A vector field  $\xi$  on a Riemannian manifold  $(M, g)$  can be thought of as a map  $\xi : M \rightarrow TM$ , where  $\pi : TM \rightarrow M$  is the tangent bundle of the manifold  $M$ . The conditions under which  $\xi$  is an isometric immersion, a totally geodesic or harmonic map, have been studied in the cases where one considers on  $TM$  the Riemannian metrics defined by Sasaki, Cheeger-Gromoll or the pseudo-Riemannian metrics of complete lift type (see [7], [16], [12], [13], [14], [11]). The conditions under which the canonical projection  $\pi : TM \rightarrow M$  is a totally geodesic or harmonic map have been also studied.

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\*partially supported by the Grant 64 /1998, Ministerul Educației Naționale, Romania

Received by the editors March 1999.

Communicated by L. Vanhecke.

1991 *Mathematics Subject Classification* : 53C07, 53C20, 58E20.

*Key words and phrases* : tangent bundle, harmonic maps, nonlinear connections, pseudo-Riemannian metrics.