On Solutions to Formal Equations

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Abstract

Let $\overline{k}$ be a field of characteristic zero equipped with an absolute value $|\cdot|$. Let $\phi_1(\mathbf{x},\mathbf{y}) = \phi_2(\mathbf{x},\mathbf{y}) = \ldots = \phi_l(\mathbf{x},\mathbf{y}) = 0$ be a system of formal power series equations in variables $\mathbf{x} = (x_1, \ldots x_n)$, $\mathbf{y} = (y_1, \ldots y_m)$ with coefficients in $\overline{k}$. The notion of $\{M_k\}$-summability of formal power series is defined relative to a sequence $\{M_k\}_{k=0}^{\infty}$ of positive real numbers. Under certain Jacobian conditions on the ϕ_i's, it is shown the $\{M_k\}$-summability of the ϕ_i's implies $\{M_k\}$-summability of any of its formal power series solutions $\mathbf{y} = \mathbf{f}(\mathbf{x})$. In particular, if the ϕ_i's are convergent, then so are its formal solutions. This result generalizes the author's earlier work on formal solutions of systems of analytic equations.

1 Introduction

It is well known that a formal power series solution of a nonzero convergent power series equation is convergent. In [11], the author proved a generalization of this result to systems of equations. A natural question, then, is what kind of properties of formal equations are preserved in their formal solutions? In this note we consider properties of systems of equations which are more general than convergence.

Let $\overline{k}$ be field of characteristic zero equipped with an absolute value $|\cdot|$. Let $\mathfrak{F}_n = \mathfrak{F}(\mathbf{x})$, $\mathbf{x} = (x_1, x_2, \ldots, x_n)$, denote the ring of formal power series in n variables with coefficients in $\overline{k}$. Let $\{M_k\}$ be a sequence of positive real numbers satisfying

$$M_k^2 \leq M_{k-1}M_{k+1}, \forall k, \quad \text{(logarithmic convexity)} \tag{1}$$

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