Characterization of translation planes by orbit lengths ii. even order

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Abstract

The translation planes of even order q^2 that admit a collineation group with point orbits at infinity of lengths $q+1$ and q^2-q are classified as either Desarguesian or Hall. Furthermore, the translation planes with spreads in $PG(3,q)$, for q even, admitting a linear collineation group with one point orbit at infinity of length $q+1$ and i point orbits at infinity of lengths $(q^2-q)/i$ for $i=1,2$ are classified as either Desarguesian, Hall, or Ott-Schaeffer.

1 Introduction.

Several years ago, the second author proposed a series of problems involving translation planes (see [15]). We mention the most difficult of these problems.

Determine the translation planes π of order q^n which admit a collineation group having an infinite point orbit of length q^n-q.

To illustrate the complexity of this problem, we note the following classes of examples satisfying the hypothesis.

First of all, when $n=2$, we, of course, have the Desarguesian and Hall planes.

Futhermore, there are tremendous varieties of generalized Hall and related planes (see Jha [16]) .

When $n=3$, there are infinitely many classes of examples including the generalized Desarguesian planes (see e.g. Jha and Johnson [17]) where the collineation group contains $GL(2,q)$.

When $n=4$, translation planes admitting $SL(2,q)\times Z_{1+q+q^2}$ correspond to Desarguesian parallelisms in $PG(3,q)$. Recently, there is an infinite class of such

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