Curves of the Projective 3–space, Tangent Developables and Partial Spreads

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1 Introduction

A *twisted cubic* $\mathcal{C}$ of $PG(3,q)$, the 3–dimensional projective space over the Galois field $GF(q)$, is given in its canonical form by

$$\mathcal{C} = \{P(t) = (t^3, t^2, t, 1), t \in GF(q) \cup \{\infty\}\},$$

where $t = \infty$ gives the point $(1,0,0,0)$. Twisted cubics over Galois fields were introduced and studied by Segre [17], [18]. Further properties were investigated by Hirschfeld [12], [13]. The main property of a twisted cubic of $PG(3,q)$ is that it is a maximal arc [10, 21.2], namely it is a set of $q+1$ points of $PG(3,q)$, no four of which are coplanar.

However, twisted cubics are also interesting because of their connection with spreads and partial spreads of $PG(3,q)$.

In $PG(3,q)$, a *spread* $\mathcal{S}$ is a set of q^2+1 lines, no two of which intersect. A *partial spread* $\mathcal{P}$ is a set of mutually skew lines, and if $|\mathcal{P}| = s$, then $\mathcal{P}$ is also called a *s–span*. Hence, a (q^2+1)–span is a spread of $PG(3,q)$.

In [3] it was shown that in $PG(3,q)$, $(q+1,3) = 1$, if $\mathcal{C}$ is a twisted cubic, then the set $\mathcal{S}$ of lines consisting of the imaginary chords of $\mathcal{C}$, the imaginary axes of the osculating developable of $\mathcal{C}$ and the tangents to $\mathcal{C}$ form a spread.

In particular, it is easily seen that the tangents to $\mathcal{C}$ form a $(q+1)$–span [10, Theorem 21.1.9] (actually the proof works for any field). For further results on twisted cubics over Galois fields see also [4].

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