KS-models and symplectic structures on total spaces of bundles

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1 Introduction

In this note we give some applications of the Félix-Thomas theorem on rational models of fibrations [FT] to the problem of constructing symplectic structures on total spaces of bundles.

It is well known that constructing symplectic structures on *closed* manifolds is an important but difficult problem in symplectic topology (see, e.g. [McDS]). One of the methods of creating new symplectic manifolds out of the known ones is based on the use of the given symplectic structures on the base and the fiber of a bundle to get a new one on the total space. There are several ways of doing this, for example, coupling forms of Lerman, Guillemin and Sternberg [GLS], fat bundles of Weinstein [TK, W] and Thurston's method [Th, McDS]. The germs of all these methods can be found in the notion of *symplectic fibration* and a theorem of Thurston which we cite below.

Definition 1.1. Let $M \xrightarrow{\pi} B$ be a locally trivial fibration with symplectic base (B, ω_B) and fibre (F, ω_F) . The fibration is called *symplectic* if its structural group acts on the fibre by symplectomorphisms. In this case each fibre $\pi^{-1}(b) = F_b$ carries a symplectic structure ω_{F_b} , [McDS, p.192].

Now the natural question arises when the total space of such a bundle admits a symplectic structure ω_M which is *compatible* with fibration π . Compatibility means

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