# Finite p-groups with few normal subgroups

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**Abstract**

This paper investigates the finite nonabelian $p$-groups $G$ with the property that every normal subgroup of $G$ either contains the commutator subgroup $G'$, or is contained in the center of $G$. We will prove that the nilpotency class of $G$ is 2 or 3 and we will find all such groups with nilpotency class 3.

## 1 Introduction

Let $G$ be a finite group. Let us denote by $\mathcal{S}(G)$ the set of the subgroups of $G$, by $\mathcal{N}(G)$ the set of the normal subgroups of $G$, by $(H]$ the ideal generated by $H$ in the lattice $(\mathcal{S}(G), \subseteq)$, and by $[H)$ the filter generated by $H$ in the same lattice. Then

$$(Z(G)] \cup [G') \subseteq \mathcal{N}(G) \subseteq \mathcal{S}(G). \tag{1}$$

If the group $G$ is abelian, then in (1) we have equalities.

If $G$ is a finite nonabelian group, then the right inclusion becomes an equality iff $G$ is one of the well-known Dedekind groups.

A natural problem is the search of the finite nonabelian groups which realize the equality in the left inequality from (1). These groups have as few normal subgroups as possible. Unfortunately, the family of these groups is too big. (It contains, for example, all the finite simple groups.) Hence, we decided to restrict ourselves to the case of the finite $p$-groups.

All the groups in discussion will be finite.

All the notation is standard.

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