Exterior sets of hyperbolic quadrics

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Abstract

Extensive studies have been made on exterior sets to hyperbolic quadrics $Q^+(2n-1,q)$ that contain exactly $(q^n-1)/(q-1)$ points. There are only few theorems on exterior sets with less than $(q^n-1)/(q-1)$ points. In this article we will prove better upper bounds for exterior sets.

1 Introduction and Basic Results

A set $\mathfrak{X}$ of points of a projective space $PG(d,q)$ (d odd) is called an *exterior set* with respect to the hyperbolic quadric $Q^+(d,q)$, if no line joining two distinct elements of $\mathfrak{X}$ has a point in common with $Q^+(d,q)$. For $d=2n-1$, we have that

$$|\mathfrak{X}| \leq \frac{q^n-1}{q-1} \quad , \tag{1}$$

because there are $(q^n-1)/(q-1)$ subspaces of dimension n that contain a fixed $(n-1)$-dimensional singular subspace and each of these subspaces can contain at most one point of $\mathfrak{X}$. By a singular subspace we mean a subspace of $PG(d,q)$ contained in $Q^+(d,q)$.

Exterior sets $\mathfrak{X}$ to $Q^+(2n-1,q)$ with $(q^n-1)/(q-1)$ points are called *maximal exterior sets (MES)*. The maximal exterior sets are completely classified (see [6], [1] and [2]).

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