Domains of $\mathbb{R}$-analytic existence in inductive limits of real separable Banach spaces

Murielle Mauer

Abstract

Every non void domain of a countable inductive limit of real separable Banach spaces is a domain of $\mathbb{R}$-analytic existence

1 Introduction

In [3], we proved that if $E = \text{ind}E_m$ is a countable inductive limit of real separable normed spaces, every non void, open and convex subset of E is a domain of analytic existence. In this note, we prove that we can drop the hypothesis of convexity if we assume that the spaces E_m are Banach spaces.

2 Result

Theorem 2.1. *If $E = \text{ind}E_m$ is an inductive limit of real separable Banach spaces, every non void domain Ω of E is a domain of analytic existence.*

To prove this result in the case of an open and convex subset Ω of E, we used that $\omega = \Omega \cap (-\Omega)$ is an open and absolutely convex subset of E such that Ω is open for its Minkowski functional p_ω. Therefore, Ω is open for one semi-norm. But by the following result and examples of J.F. Colombeau and J. Mujica (cf. Lemma 3.5 in [1] and Example 3.1 in [2]), every arbitrary domain Ω in such a space E is open for one semi-norm.

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