Infinitary axiomatizability of slender and cotorsion-free groups

Saharon Shelah [*] Oren Kolman [†]

Abstract

The classes of slender and cotorsion-free abelian groups are axiomatizable in the infinitary logics $L_{\infty\omega_1}$ and $L_{\infty\omega}$ respectively. The Baer-Specker group $\mathbb{Z}^\omega$ is not $L_{\infty\omega_1}$-equivalent to a slender group.

1 Introduction

In 1974, Paul Eklof [4] used infinitary logic to generalize some classical theorems of infinite abelian group theory. He characterized the strongly $\aleph_1$-free groups as exactly those abelian groups which are $L_{\infty\omega_1}$-equivalent to free abelian groups, used his criterion to deduce that the class of free abelian groups is not $L_{\infty\omega_1}$-definable, and showed that the Baer-Specker group $\mathbb{Z}^\omega$ is not $L_{\infty\omega_1}$-equivalent to a free abelian group, strengthening a theorem of Baer [1] that $\mathbb{Z}^\omega$ is not free. This paper continues in the tradition allying infinite abelian group theory with infinitary logic. Its main result is the following:

Theorem 1.1. *The class* **SL** *of slender abelian groups is axiomatizable in the infinitary logic* $L_{\infty\omega_1}$.

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