On nonlinear hyperbolic problems with nonlinear boundary feedback

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Abstract

In this paper we prove the existence, uniqueness and uniform decay of strong and weak solutions of the nonlinear model of the wave equation

$$u_{tt} - \Delta u + f(u) + h(\nabla u) = 0$$

in bounded domains with nonlinear dissipative boundary conditions given by

$$\frac{\partial u}{\partial \nu} + g(u_t) = 0.$$

The existence is proved by means of Faedo-Galerkin method and the asymptotic behavior is obtained making use of the multiplier technique due to Komornik and Zuazua .

1 Introduction

Consider the nonlinear wave equation with a nonlinear boundary dissipative term

$$(*) \qquad \begin{cases} u_{tt} - \Delta u + f(u) + h(\nabla u) = 0 \quad \text{in} \quad \Omega \times (0, \infty), \\ u = 0 \quad \text{on} \quad \Gamma_1 \times (0, \infty), \\ \dfrac{\partial u}{\partial \nu} + g(u_t) = 0 \quad \text{on} \quad \Gamma_0 \times (0, \infty), \\ u(x, 0) = u^0(x); \quad u_t(x, 0) = u^1(x) \quad \text{in} \quad \Omega, \end{cases}$$

where Ω is a bounded domain of $\mathbf{R}^n$, $n \geq 1$, with a smooth boundary $\Gamma = \Gamma_0 \cup \Gamma_1$. Here, Γ_0 and Γ_1 are closed and disjoint , ν represents the unit outward normal

Received by the editors March 1999.
Communicated by P. Laubin.