Weighted eigenfunctions and Gauss curvature of conical revolution surfaces

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Abstract

We give a description of Gauss curvatures in revolution surfaces with conical singularities at the extreme opposite points thanks to positive eigenfunctions of an eigenvalue problem in dimension one with a prescribed singular weight.

1 Introduction

Given a revolution surface

$$S = \{(\alpha(v)\cos u, \alpha(v)\sin u, \beta(v)); 0 < u < 2\pi,\ a < v < b\} \tag{1}$$

where $\alpha(v) > 0, \alpha, \beta$ regular functions and supposing the generating curve $\gamma = (\alpha(v), 0, \beta(v))$ parametrized by arc-length, that is

$$\alpha'^2 + \beta'^2 = 1 \text{ in }]a, b[$$

Then the Gauss curvature K of S is given by

$$K = \frac{-\alpha''(v)}{\alpha(v)},\ v \in]a, b[\tag{2}$$

[DC, p. 162].

If α, β are regular up to $[a, b]$ and

$$\begin{cases} \alpha(a) = \alpha(b) = 0,\ 0 < \alpha'(a) \leq 1,\ -1 \leq \alpha'(b) < 0 \\ \beta(a) < \beta(b) \end{cases} \tag{3}$$

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