A note on nonexistence of global solutions to a nonlinear integral equation

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Abstract

In this paper we study the Cauchy problem for the integral equation

\[ u_t = (-\Delta)^\beta u + h(t)u^{1+\alpha} \quad \text{in } \mathbb{R}^N \times (0, T), \]

where \( 0 < \beta \leq 2 \). We obtain some extension of results of Fujita who considered the case \( \beta = 2 \) and \( h \equiv 1 \).

1 Introduction

This article deals with the blow-up of positive solutions to the Cauchy problem for the integrodifferential equation

\[ u_t = (-\Delta)^\beta u + h(t)u^{1+\alpha} \quad \text{in } \mathbb{R}^N \times (0, T), \]

\[ u(x, 0) = u_0(x) \geq 0 \quad \text{for } x \in \mathbb{R}^N, \]

where \((-\Delta)^\beta\), for \( 0 < \beta \leq 2 \), denote the fractional power of the operator \(-\Delta\). It is assumed that \( u_0 \) is a continuous function defined on \( \mathbb{R}^N \) and \( \alpha \) is a positive constant. The function \( h \) satisfies

\[ h_1) \quad h \in C[0, \infty), h \geq 0, \]

\[ h_2) \quad c_0 t^\sigma \leq h(t) \leq c_1 t^\sigma \quad \text{for sufficiently large } t, \quad \text{where } c_0, c_1 > 0 \quad \text{and } \sigma > -1 \text{ are constants.} \]