On Improving Uspensky-Sherman's Normal Approximation by an Edgeworth-Expansion Approximation

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Abstract

The exact bound of the remainder in normal approximation obtained by Uspensky (1937) was sharpened by Sherman (1971) for the sample mean from a continuous uniform distribution. Their exact bounds of $O(n^{-1})$ is now improved to an exact bound of $O(n^{-2})$ on the remainder after one-step higher-order Edgeworth-expansion approximation. The estimations of the error obtained from the improved bound is so sharp that it may provide practically useful information in statistical applications.

1 Introduction.

Let $X_1, X_2, \cdots, X_n$ be i.i.d. (independently and identically distributed) rv's (random variables) having mean zero and finite variance, i.e., $EX^2 \equiv \sigma^2 < \infty$. We consider the normalized sample mean $T_n = (\sigma/\sqrt{n})^{-1}\bar{X}_n$, where $\bar{X}_n = n^{-1}\sum_{j=1}^{n} X_j$. Denote its cdf (cumulative distribution function) and the standard normal cdf, by $F_n(x) = P(T_n \leq x)$ and $\Phi(x)$, respectively; and put $\Delta_n = \sup_{x\in\Re} |F_n(x) - \Phi(x)|$. Then, $\lim_{n\to\infty} \Delta_n = 0$ by the well-known CLT. This normal approximation is frequently used in statistical applications and it has been justified typically by the reference to the CLT (Central Limit Theorem). Unfortunately, this simple asymptotic

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