

The Cauchy integral formulas on the octonions

Xingmin Li Lihong Peng*

Abstract

As the last one of the finite, alternative, division algebra, the Cayley-Graves algebra or the octonion algebra \mathbf{O} , is a non-commutative, non-associative division algebra, in which the analysis problems that would be a direct generalization of the complex analysis and the quaternion analysis, have been studied systematically. Taking the associator as a measure, the Cauchy integral formulas, the Cauchy theorems and the inverse theorems of the Cauchy integral formulas are obtained on the octonions. Some applications are also given.

1 Introduction

It is well-known that [J], the only finite dimensional alternative division algebras over \mathbf{R} are

- a) Real algebra \mathbf{R} ;
- b) Complex algebra \mathbf{C} ;
- c) Quaternion algebra \mathbf{H} ;
- d) Octonion algebra \mathbf{O} ;

with the embedding relations: $\mathbf{R} \subset \mathbf{C} \subset \mathbf{H} \subset \mathbf{O}$.

\mathbf{R} and \mathbf{C} are commutative and associative, \mathbf{H} is associative but not commutative, while \mathbf{O} is neither commutative nor associative.

Quaternions were invented by the Irish mathematician W. R. Hamilton in 1843 after a lengthy struggle to extend the theory of complex numbers to three dimensions. Rejecting the commutative law he got the quaternions. Quaternions have

*Research supported by NNSF of China (Grant No: 19631080 and 69735020)

Received by the editors august 2000.

Communicated by R. Delanghe.

1991 *Mathematics Subject Classification* : 30G35, 17A35.

Key words and phrases : octonion, non-commutative, non-associative, division algebra, \mathbf{O} -analytic function, associator, Cauchy integral formulas.