Fixed point results for compact maps on closed subsets of Fréchet spaces and applications to differential and integral equations

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Abstract

In this paper, we establish fixed point results for compact maps $f : X \to \mathbb{E}$ defined on arbitrary closed subsets X of a Fréchet space $\mathbb{E}$. In particular, we obtain a continuation principle for suitable compact homotopy $h : X \times [0,1] \to \mathbb{E}$. Afterwards, those results are applied to differential equations and to Fredholm integral equations on the real line.

1 Introduction

It is well known (see [14]) that if $h : X \times [0,1] \to \mathbb{E}$ is a compact map defined on X the closure of an open set of a locally convex space $\mathbb{E}$, and if $h(x,0) \equiv \hat{x} \in \mathrm{int}(X)$, then one of the following statements holds:

(a) $h(\cdot,1)$ has a fixed point;

(b) there exist $\lambda \in (0,1)$ and $x \in \partial X$ such that $x = h(x,\lambda)$.

In the particular case where $\mathbb{E}$ is a Banach space, this important result was widely applied, notably to nonlinear differential equations. Unfortunately, very few applications were given in the case where $\mathbb{E}$ is a locally convex space which is not normable. The problem is that in many potential applications, the appropriate set X to work with has empty interior, see for example [4].

*This work was partially supported by CRSNG Canada
Received by the editors February 2001.
Communicated by J. Mawhin.
1991 *Mathematics Subject Classification* : 47H10,34A12,45B05.