On the asymptotic behaviour of the solutions of $-(r(t)u')' + p(t)u = 0$, where p is not of constant sign *

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1 Introduction

We consider a differential equation of the form

$$Lu = -(r(t)u')' + p(t)u = 0,$$

where r is a (strictly) positive continuous function on an open interval $]a, b[$ in the real numbers $\mathbb{R}$ ($a = -\infty$ and/or $b = +\infty$ are allowed) and p is a locally integrable function on $]a, b[$. In [9] and [1], we can find a complete study of the asymptotic behaviour of the solutions of $Lu = 0$, in the neighborhood of b, in the special case when $p \geq 0$. In [2], we can find a similar study for the case when $p \leq 0$. Note that in these papers, the authors assume that p is continuous, does not vanish (in the neighborhood of b) and that $b = +\infty$, but it is easily seen that most of their results remain true in the more general situation considered here. We just point out that we cannot use the "duality principle" of [1] and [2], because our hypotheses on $1/r$ and p are not symmetric. Nevertheless, when a result of [1] or [2] is proved by means of this principle, it is always possible to give a direct proof avoiding this principle.

Some results on the asymptotic behaviour of solutions are also known when p is not necessarily of constant sign (see [7], XI, 9 or [3], for example).

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