Submanifolds in a hyperbolic space form with flat normal bundle

Liu Ximin

Abstract

In this paper we give some rigidity results for compact submanifolds in a hyperbolic space form with flat normal bundle to be totally umbilical.

1 Introduction

Let $M^{n+p}(c)$ be an $(n+p)$-dimensional Riemannian manifold with constant sectional curvature c. We also call it a space form. When $c > 0$, $M^{n+p}(c) = S^{n+p}(c)$ (i.e. $(n+p)$-dimensional sphere space); when $c = 0$, $M^{n+p}(c) = R^{n+p}$ (i.e. $(n+p)$-dimensional Euclidean space); when $c < 0$, $M^{n+p}(c) = H^{n+p}(c)$ (i.e. $(n+p)$-dimensional hyperbolic space). We simply denote $H^{n+p}(-1)$ by H^{n+p}. Let M^n be an n-dimensional submanifold in $M^{n+p}(c)$. As it is well known, there are many rigidity results for minimal submanifolds or submanifolds with constant mean curvature H in $M^{n+p}(c)$ $(c \geq 0)$ by use of J. Simons' method, for example, see [1], [4], [7], [12], etc., but less of that were obtained for submanifolds immersed into a hyperbolic space from. Walter [13] gave a classification for non-negatively curved compact hypersurfaces in a space form under the assumption that the rth mean curvature is constant. Morvan-Wu [6], Wu [14] also proved some rigidity theorems for complete hypersurfaces M^n in a hyperbolic space form $H^{n+1}(c)$ under the assumption that the mean curvature is constant and the Ricci curvature is non-negative. Moreover, they proved that M^n is a geodesic distance sphere in $H^{n+1}(c)$ provided that it is compact.

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