Amenability of certain Banach algebras with application to measure algebras on foundation semigroups

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A well-known result of G. Willis asserts that for a locally compact group G if the group algebra $L^1(G)$ is separable then $L^1(G)$ is amenable if and only if $I_0(L^1(G))$ is the unique maximal ideal of $\mathcal{F}_a$, where

$$I_0(L^1(G)) = \{f \in L^1(G) : \int_G f = 0\},$$

$$\mathcal{F}_a = \{J_\mu : \mu \text{ is a probability measure on } G\},$$

and

$$J_\mu = \{f - f * \mu : f \in L^1(G)\}^-,$$

(see theorem 1.2 of [**11**]). It is also proved in proposition 1.3 of [**11**] that if $L^1(G)$ is separable and amenable, then there is a discrete probability measure μ on G such that $I_0(L^1(G)) = J_\mu$.

The aim of the present paper is to extend the first result to the general setting of separable Lau algebras and the second result to an extensive class of topological semigroups, namely foundation semigroups. It should be noted that $L^1(G)$, the Fourier algebra $A(G)$, the Fourier-Stiltjes algebra $B(G)$ of a locally compact group G, and the measure algebra $M_a(S)$ of a topological semigroup S are elementary examples of Lau algebras. The class of foundation semigroups is extensive, and includes all discrete semigroups, all locally non-locally-null subsemigroups of locally compact groups and those subsemigroups related to those considered by Rothman [**9**]. For many other examples, see [**10**, Appendix B].

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