Decomposition of a numerical semigroup as an intersection of irreducible numerical semigroups *

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Abstract

Every numerical semigroup S admits a decomposition $S = S_1 \cap \cdots \cap S_n$ with S_i irreducible (that is, S_i is symmetric or pseudo-symmetric) for all i. We give lower and upper bounds for the minimal number of irreducibles in such a decomposition. We also study the problem of determining those numerical semigroups for which all S_i are symmetric, and when all S_i are pseudo-symmetric. We introduce and characterize the concept of atomic numerical semigroup.

1 Introduction

A **numerical semigroup** is a subset S of $\mathbb{N}$ closed under addition, it contains the zero element and generates $\mathbb{Z}$ as a group (here $\mathbb{N}$ and $\mathbb{Z}$ denote the set nonnegative integers and the set of the integers, respectively). From (see [2] or [10]) we know that the set $\mathbb{N} \setminus S$ is finite. We refer to the greatest integer not belonging to S as the **Frobenius number** of S and denote it by $g(S)$.

We say that a numerical semigroup is **irreducible** if it can not be expressed as an intersection of two numerical semigroups containing it properly. In [7] it is show that S is irreducible if and only if S is maximal in the set of all numerical semigroups with Frobenius number $g(S)$. From [2] and [4] we can deduce that the class

*Special thanks to P. A. García-Sánchez for his comments and suggestions.
†The first author has been supported by the project BFM2000-1469.
‡The second author has been supported by the University of Évora.
Received by the editors June 2001.
Communicated by M. Van den Bergh.
2000 *Mathematics Subject Classification :* 20M14, 20M30, 13H10.
Key words and phrases : numerical semigroup, symmetric and pseudo-symmetric numerical semigroup, irreducible numerical semigroup, Frobenius number.