On the selection of basic orthogonal sequences in non-archimedean metrizable locally convex spaces

Wiesław Śliwa

Abstract

Our main result (Theorem 1) follows that any infinite-dimensional subspace F of a non-archimedean metrizable locally convex space E with an orthogonal basis (e_n) contains a basic orthogonal sequence equivalent to a block basic orthogonal sequence relative to (e_n) (Proposition 2). Hence any infinite-dimensional non-archimedean metrizable locally convex space F possesses a basic orthogonal sequence equivalent to a block basic orthogonal sequence relative to an orthogonal basis in $c_0^{\mathbb{N}}$ (Corollary 3).

Introduction

In this paper all linear spaces are over a non-archimedean non-trivially valued field $\mathbb{K}$ which is complete under the metric induced by the valuation $|\cdot| : \mathbb{K} \to [0, \infty)$. For fundamentals of locally convex Hausdorff spaces (lcs) and normed spaces we refer to [2], [4] and [3]. Orthogonal bases and basic orthogonal sequences in locally convex spaces are studied in [1], [6] and [8].

Any infinite-dimensional Banach space of countable type is isomorphic to the Banach space c_0 of all sequences in $\mathbb{K}$ converging to zero (with the sup-norm) ([3], Theorem 3.16), so it has an orthogonal basis.

There exist infinite-dimensional Fréchet spaces of countable type without a Schauder basis (see [7]). Nevertheless, any infinite-dimensional metrizable lcs E of finite

Received by the editors October 2001.
Communicated by F. Bastin.
1991 *Mathematics Subject Classification :* 46S10, 46A35.
Key words and phrases : Orthogonal bases, block basic orthogonal sequences.