Newton Polyhedra and the Poles of Igusa's Local Zeta Function

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Abstract

We give a very explicit formula for Igusa's local zeta function $Z_f(s, \chi)$ associated to a polynomial f in several variables over the p-adic numbers and to a character χ of the units of the p-adic integers (with conductor 1). This formula holds when f is sufficiently non-degenerated with respect to its Newton polyhedron $\Gamma(f)$. Using this formula, we give a set of possible poles of $Z_f(s, \chi)$, together with upper bounds for their orders. Moreover this formula implies that $Z_f(s) = Z_f(s, \chi_{\text{triv}})$ has always at least one real pole.

1 Introduction

For p prime, denote the field of p-adic numbers by $\mathbb{Q}_p$, the ring of p-adic integers by $\mathbb{Z}_p$, and the finite field of p elements by $\mathbb{F}_p$. If R is a commutative ring with identity, we will denote the set of its units by $R^\times$.

Definition 1.1. *Let $f(x) = f(x_1, \ldots, x_n) \in \mathbb{Z}_p[x_1, \ldots, x_n]$ with p prime. For $z \in \mathbb{Q}_p$, $\operatorname{ord} z \in \mathbb{Z} \cup \{\infty\}$ denotes the valuation, $|z| = p^{-\operatorname{ord} z}$ and $\operatorname{ac}(z) = p^{-\operatorname{ord} z} z$ denotes the angular component. Let $\chi : \mathbb{Z}_p^\times \to \mathbb{C}^\times$ be a character of $\mathbb{Z}_p^\times$, i.e., a group homomorphism with finite image. We formally put $\chi(0) = 0$. To the above data we associate the following two* Igusa local zeta functions *(the global and the local one):*

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