Extremal Kähler $\mathcal{A}C^{\perp}$-surfaces

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Abstract

The aim of this paper is to give an example of a Kähler extremal metric with harmonic anti-self-dual Weyl tensor on the Hirzebruch surface F_1.

Introduction.

It is known that self-dual Kähler 4-manifolds (M, g, J) are Bochner-flat. M. Matsumoto and S. Tanno proved [M-T] that every Bochner flat Kähler manifold satisfies the condition

$$\nabla_X \rho(Y, Z) = \frac{1}{(2\mathrm{dim}M + 4)}(g(X, Y)Z\tau + g(X, Z)Y\tau + 2g(Y, Z)X\tau \qquad (*)$$
$$-g(JX, Y)(JZ)\tau - g(JX, Z)(JY)\tau),$$

where τ is the scalar curvature of (M, g). Consequently, Ricci tensor ρ of any Kähler Bochner-flat manifold satisfies the condition

$$\nabla_X \rho(X, X) = \frac{2}{n+2} X\tau g(X, X), \qquad (**)$$

where τ is the scalar curvature of (M, g) and $n =$dim$M$. This property was studied by A. Gray in [G]. A. Gray called Riemannian manifolds satisfying $(**)$ the $\mathcal{A}C^{\perp}$ manifolds. In [J-1] we showed that every Kähler surface has a harmonic anti-self-dual part W^- of the Weyl tensor W (i.e. such that $\delta W^- = 0$) if and only if it is an $\mathcal{A}C^{\perp}$-manifold. We also proved that a Kähler manifold is an $\mathcal{A}C^{\perp}$-manifold if and only

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