A Groebner Basis Algorithm for Computing the Rational L.-S. Category of Elliptic Pure Spaces

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Abstract

The rational Lusternik-Schirelmann category, $\mathrm{cat}_0(S)$, of an elliptic space S has been characterized in terms of its Sullivan minimal model $(\Lambda V, d)$ as $\mathrm{cat}_0(S) = \sup\{k \mid \exists w \in \Lambda^{\geq k}V, [w] \text{ is a top class}\}$. We combine a method for computing a representative of the fundamental class of any elliptic space with a Groebner basis approach so that, for S a pure elliptic space, reduction of this representative provides one that achieves the $\mathrm{cat}_0(\Lambda V, d)$ upper bound.

Introduction

The *Lusternik-Schirelmann category* [9, 13], cat S, of a topological space S is the least integer m such that S is the union of $m+1$ open sets, each contractible in S. For S a simply connected CW complex, the *rational L.-S. category*, $\mathrm{cat}_0(S)$, introduced by Felix and Halperin in [5] is given by $\mathrm{cat}_0(S) = \mathrm{cat}(S_{\mathbb{Q}}) \leq \mathrm{cat}\,(S)$.

Recently [6], the rational L.-S. category of an elliptic space S has been characterized in terms of its minimal model $(\Lambda V, d)$ as $\mathrm{cat}_0(S) = \sup\{k \mid \exists w \in \Lambda^{\geq k}V, [w] \text{ is a top class}\}$. We combine a method [12] for computing a representative of the fundamental class of any elliptic space with a Groebner basis approach so that, for a pure elliptic space, reduction of this representative yields one that achieves the $\mathrm{cat}_0(\Lambda V, d)$ upper bound.

In this paper, all spaces are CW-complexes that are simply connected and whose rational homology is finite dimensional in each degree.

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