Harmonic-Killing vector fields*

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Abstract

In this paper we consider the harmonicity of the 1-parameter group of local infinitesimal transformations associated to a vector field on a (pseudo-) Riemannian manifold, to study this class of vector fields, which we call harmonic-Killing vector fields.

1 Introduction

Different properties have been considered for the integral flows corresponding to vector fields. For instance, when the corresponding 1-parameter group of local transformations consists of isometric, affine or conformal maps, a vector field is called respectively Killing, affine-Killing or conformal. However, harmonicity has only been used to study other aspects of vector fields. In [13] harmonic vector fields are defined as those having harmonic associated 1-form. Several authors ([6], [10]) use the harmonicity of the section induced on the tangent bundle with different lift metrics: Sasaki, complete, ...

We introduced the term 1-*harmonic-Killing* vector field for the case when the transformations have zero linear part of their tension field, which [9] had referred to as harmonic infinitesimal transformations. The approach emphasizes the importance of the complete lift metric for tangent bundles in the study of harmonicity. We point out that a vector field is a Jacobi field along the identity map if and only if it is a 1-harmonic-Killing vector field.

Given a 1-harmonic-Killing vector field, X, on a (semi-)Riemannian manifold (M, g) and a parallel $(1,1)$-tensor field, T, we use the definition of harmonic $(1,1)$-tensor field ([6]), as a harmonic map from (TM, g^C) to itself, to show that TX is

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