Multary epistasis

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Abstract

In this note, we introduce and study the notion of normalized epistasis of a fitness function over not necessary binary alphabets, as an indicator of its GA-hardness. Fitness functions with minimal and maximal normalized epistasis are explicitly described.

Introduction

The classical genetic algorithm (GA) starts from a positive real-valued function f on $\Omega = \{0,1\}^\ell$ (the set of all length ℓ strings $s = s_{\ell-1} \dots s_0$), whose maximum (or minimum) we want to find. It has long been understood (in particular through examples given in [3, et al]) that linkage between bits may make it hard for the GA to find the maximum of f. In [5] Rawlins compares this phenomenon to the analogous situation in genetics, where a gene at some locus in the chromosome may hide the (phenotypical) effect of another gene at a different locus, cf. [6]. When this phenomenon occurs, one refers to the first gene as being *epistatic* to the second one.

Adapting this idea to the framework of GAs, Rawlins thus speaks of *minimal* (or *zero*) *epistasis*, when every bit is independent of any other one, i.e., if the fitness function f may be given as

$$f(s_{\ell-1} \dots s_0) = \sum_{i=0}^{\ell-1} g(i, s_i).$$

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