Retracting spreads

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Abstract

The concept of spread-retraction is introduced by which certain spreads in $PG(4m+1, q)$ or $PG(4m-1, q)$ may be 'retracted' to either Baer subgeometry partitions of $PG(2m, q^2)$ or to mixed partitions of $PG(2m-1, q^2)$ respectively. This characterizes the spreads produced by such partitions abstractly and furthermore allows a vast number of new mixed partitions to be recognized.

1 Introduction

In this article, we shall be discussing partitions of the points of finite projective geometries Σ over $GF(q^2)$. When Σ is isomorphic to $PG(2m, q^2)$, the partition components are Baer subgeometries isomorphic to $PG(2m, q)$. When Σ is isomorphic to $PG(2n-1, q^2)$, it is possible to have a so-called 'mixed' partition of $\beta\ PG(n-1, q^2)'s$ and $\alpha\ PG(2n-1, q)'s$. The configuration is such that $\alpha(q+1) + \beta = q^{2n} + 1$.

The interest in such partitions lies in the fact they may be used to construct spreads and hence translation planes. Baer subgeometry partitions produce translation planes of order q^{2m+1} where mixed partitions produce translation planes of order q^{2n}. These constructions are applications of the theory of Segré varieties and are given in Hirschfeld and Thas [8], p. 206. In particular, all Baer subgeometries produce $GF(q)$-reguli in the associated spread. When the partition is a Baer subgeometry partition, the spread is a union of mutually disjoint $GF(q)$-reguli. Furthermore, mixed partitions of $PG(2m-1, q^2)$ by $PG(m-1, q^2)'s$ and $PG(2m-1, q)'s$ produce spreads in $PG(4m-1, q)$ which contain $d\ GF(q)$-reguli provided there are $d\ PG(2m-1, q)'s$ in the mixed partition.

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