On dual Euler-Simpson formulae

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Abstract

The dual Euler-Simpson formulae are given. A number of inequalities, for functions whose derivatives are either functions of bounded variation or Lipschitzian functions or functions in L_p-spaces, are proved. The results are applied to obtain the error estimates for some quadrature rules..

1 Introduction

One of the elementary quadrature rules of closed type is the Simpson's rule based on the Simpson's formula [4, p. 45]

$$\int_a^b f(t)\mathrm{d}t = \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right] - \frac{(b-a)^5}{2880} f^{(4)}(\xi), \tag{1.1}$$

where $a \le \xi \le b$. A simple quadrature rule of open type, which is closely related to the Simpson's rule, is based on the following three-point formula [4, p. 71]

$$\begin{aligned}\int_a^b f(t)\mathrm{d}t &= \frac{b-a}{3}\left[2f\left(\frac{3a+b}{4}\right) - f\left(\frac{a+b}{2}\right) + 2f\left(\frac{a+3b}{4}\right)\right] \\ &\quad + \frac{7(b-a)^5}{23040} f^{(4)}(\eta),\end{aligned} \tag{1.2}$$

where $a \le \eta \le b$. The formulae (1.1) and (1.2) are valid for any function f which has a continuous fourth derivative $f^{(4)}$ on $[a,b]$. P. S. Bullen [3] proved that, under certain convexity assumptions on f, the three-point quadrature rule based on the

Received by the editors September 2000.
Communicated by J. Mawhin.
1991 *Mathematics Subject Classification :* 26D15, 26D20, 26D99.
Key words and phrases : Dual Euler-Simpson formulae, quadrature formulae, functions of bounded variation, Lipschitzian functions.