Solutions to equations of p-Laplacian type in Lorentz spaces

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Abstract

We find solutions to the linear problem (1.1) and to the p-Laplacian type problem (1.2) in Lorentz spaces, improving the sumability of the solutions.

1 Introduction

We consider the linear problem

$$(1.1)\begin{cases} L(u) \equiv \operatorname{div}(M(x)\nabla u) & = & \operatorname{div} F & \text{in} & \Omega \\ u & = & 0 & \text{in} & \partial\Omega \end{cases}$$

where $\Omega \subset R^N$ is a bounded domain and $M(x)$ is a symmetric matrix in $L^\infty(\Omega)^{N\times N}$ satisfying the ellipticity condition: $M(x)\xi \cdot \xi \geq \alpha|\xi|^2$ for $x \in \Omega$, $\xi \in \mathbb{R}^N$ $(\alpha > 0)$, and the nonlinear problem

$$(1.2)\begin{cases} N(u) \equiv \operatorname{div}(a(x,u(x),\nabla u(x))) & = & \operatorname{div} F & \text{in} & \Omega \\ u & = & 0 & \text{in} & \partial\Omega \end{cases}$$

Specifically, let $A(u)$ be a monotone operator of Leray-Lions type ([LL],[Li]) : $A(u) = \operatorname{div}(a(x,u,\nabla u))$, with $a : \Omega \times \mathbb{R} \times \mathbb{R}^N \to \mathbb{R}^N$ a Caratheodory function verifying the following conditions:

i)There exist two constants $\alpha, \beta > 0$, and a function $d(x)$ in $L^{p'}(\Omega)$ $(\frac{1}{p} + \frac{1}{p'} = 1)$, $1 < p < N$ such that:

$$a(x,s,\xi)\xi \geq \alpha|\xi|^p \quad (1.3)$$

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