The fundamental class of a rational space, the graph coloring problem and other classical decision problems

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Abstract

The problem of k-coloring a graph is equivalent to deciding whether a particular cohomology class of a certain rational space vanishes. Although this problem is NP-hard we are able to construct a fast (polynomial) algorithm to give a representative of this class. We also associate to other classical decision problems rational spaces so that the given problem has a solution if and only if the associated space is not elliptic. As these spaces have null Euler homotopy characteristic we easily characterize when the given problem has a solution in terms of commutative algebra.

1 Introduction

In [14] the authors associate to a given graph G and any integer $k \geq 2$ a rational space $S_{G,k}$ and prove that the graph can be k-colored if and only if the singular cohomology of $S_{G,k}$ with rational coefficients is infinite dimensional. On the other hand, in [15], the second author gives an explicit formula for a cohomology class of the formal dimension of certain spaces (the so called finite pure spaces) in such a way that the non vanishing of this class is equivalent to the finiteness of the rational cohomology of the space. As the spaces $S_{G,k}$ are pure, the problem of k-coloring a graph is equivalent to determining when a particular cohomology class vanishes (let us call it the fundamental class).

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