Proof of a conjecture on word complexity

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Abstract

An integer n is k-reachable if there exists a word of length k which contains exactly n non-empty different factors. Given k, all the k-reachable integers are between k and $\frac{k(k+1)}{2}$ but, between these two values, not all the integers are k-reachable. We give a general construction which associates to each k a family of words containing for each k-reachable integer n, exactly one word having n different factors. This also proves the conjecture of Kása about the smallest number m_k such that all the integers between m_k and $\frac{k(k+1)}{2}$ are k-reachable.

Résumé

Un entier n est k-atteignable s'il existe un mot de longueur k contenant exactement n facteurs non vides différents. Étant donné k, tous les entiers k-atteignables sont compris entre k et $\frac{k(k+1)}{2}$; mais entre ces deux valeurs, tous les entiers ne sont pas k-atteignables. Nous donnons une construction générale pour associer à chaque k une famille de mots qui, pour tout entier k-atteignable n, contient exactement un mot ayant n facteurs différents. Cette construction nous permet également de prouver la conjecture de Kása concernant le plus petit nombre m_k tel que tous les entiers compris entre m_k et $\frac{k(k+1)}{2}$ sont k-atteignables.

1 Introduction

The combinatorial properties of words have been studied intensively since the 60-70's, although the first papers in this area are those of Thue at the beginning of the century [6, 7] (see [4, 5] for a general overview and a large bibliography). However, most of the works deal with infinite words. Less attention has been given to finite sequences (for more informations see, e.g., [1]).

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