On Infinitesimally k-Flat Homogeneous Spaces

Jürgen Berndt* Evangelia Samiou

1 Introduction

A k-flat in a Riemannian manifold M is a k-dimensional, totally geodesic, complete, connected, flat submanifold. A homogeneous Riemannian manifold M is said to be k-flat homogeneous if every geodesic in M lies in a k-flat and if the isometry group of M acts transitively on the set of pairs (p, T), where T is a k-flat in M and $p \in T$. A well-known result by Tits and Wang says that a 1-flat homogeneous space, or equivalently a two-point homogeneous space, is symmetric (for an elegant proof see [6]). This was generalized for arbitrary $k \geq 2$ to k-flat homogeneous spaces by Heintze, Palais, Terng and Thorbergsson in [2] for the compact case and by the second author in [3] and [4] for the general case. In this paper we investigate in how far these results are infinitesimal phenomena.

An infinitesimal curvature model (V, g, R) consists of a finite-dimensional real vector space V, a positive definite inner product g on V, and an algebraic curvature tensor R. An infinitesimal k-flat in (V, g, R) is a k-dimensional linear subspace F of V such that $R(X, Y)Z = 0$ for all $X, Y, Z \in F$. Let $\mathcal{A}$ be the group of automorphisms of g and R, i.e. the isometries A of (V, g) satisfying $R(AX, AY)AZ = AR(X, Y)Z$ for all $X, Y, Z \in V$. We say that (V, g, R) is infinitesimally k-flat homogeneous if every one-dimensional linear subspace of V is contained in an infinitesimal k-flat in (V, g, R) and if $\mathcal{A}$ acts transitively on the set of infinitesimal k-flats in (V, g, R). A Riemannian manifold M with metric g and curvature tensor R is said to be infinitesimally k-flat homogeneous if for every $p \in M$ the infinitesimal curvature model (T_pM, g_p, R_p) is infinitesimally k-flat homogeneous.

*Research supported by EPSRC Grant GR/M18355

Received by the editors May 2000.

Communicated by L. Vanhecke.

2000 *Mathematics Subject Classification* : 53C24, 53C25.

Key words and phrases : k-flat rigidity, infinitesimal flats, semi-symmetric spaces, pointwise Osserman spaces, two-point homogeneous spaces, cones.