Positive solutions for a nonlocal boundary-value problem with vector-valued response

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1 Introduction

We investigate the nonlinear problem:

$$\frac{d}{dt}(k(t)x'(t)) + V_x(t, x(t)) = 0, \text{ a.e. in } [\mathbf{0}, \mathbf{T}] \tag{1.1}$$

$$x(0) = 0,$$

where

H *$T > 0$ is arbitrary, $V : \mathbf{R} \times \mathbf{R}^n \to \mathbf{R}$ is Gateaux differentiable in the second variable and measurable in t function, $k : [\mathbf{0}, \mathbf{T}] \to \mathbf{R}^+$, $x = (x_1, ..., x_n)$.*

We are looking for solutions of (1.1) being a pair (x, p) of absolutely continuous functions $x, p : [0, T] \longrightarrow \mathbf{R}^n$, $x(0) = 0$ such that

$$\frac{d}{dt}p(t) + V_x(t, x(t)) = 0,$$
$$p(t) = k(t)x'(t)).$$

Of course, if k is an absolutely continuous function, then our solution of (1.1) belongs to $C^{1,+}([0, T], \mathbf{R}^n)$ of continuously differentiable functions x whose derivatives x' are absolutely continuous. In the sequel we will not assume that V_x is superlinear

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