The index of certain hyperelliptic curves over p-adic fields*

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1 Introduction

Throughout k will be a local field of characteristic zero, i.e., k will be a finite extension of the field of p-adic numbers $\mathbb{Q}_p$. O_k will be the ring of integers in k, κ its residue field, and π a fixed uniformizing element, and v the corresponding valuation.

Let C be a geometrically connected smooth projective curve over k, the index of C, $I(C)$, is the greatest common divisor of the degrees of the divisors on C. For curves over local fields other interpretations of the index exist. For instance an important theorem of Roquette and Lichtenbaum (cf. [6]) tells us that

$$I(C) = \#[\ker(Br(k) \to Br(k(C))] \qquad (RL)$$

with $Br(k), Br(k(C))$ the Brauer groups of respectively k and $k(C)$.

Since the existence of a k-rational point implies clearly that the index is 1, the determination of the index of a curve C is related to the basic diophantine question whether or not the curve C has a k-rational point.

We like to determine the index of curves C defined by an affine equation of the form $Y^2 = h(X)$, with $h(X) \in k[X]$. There is always a rational point on such curves in some quadratic extension of k, so for such curves the index is necessarily 1 or 2. (This fact follows also from the other characterization, (RL), of the index. Namely $k(C) = k(X)(\sqrt{h(X)})$ so the kernel of $Br(k) \to Br(k(C))$ consists of quaternion algebras only. And there is only one quaternion division algebra over the local field

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