BINOMIAL BOWLING

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The game of bowling provides an inherent mathematical structure that can be analyzed under various probabilistic scenarios. Although there is not much randomness involved for skilled bowlers, novices may feel that they are simply knocking down pins at random. In fact, suppose we play by knocking down anywhere from 0 to 10 pins at random, with each number of pins being equally likely, and then roll a second ball that also knocks down pins at random from the remainder. Then the average score over ten frames would be about 91.4127 [2], which is not a bad score for beginners. But this average differs from the mean score of all possible games when the scores are weighted according to their frequencies. In [1], it is shown that there are approximately 5.7 billion billion possible ways to bowl a game and that the average score is about 80.

In this article, we shall consider another scenario in which the pins are knocked down according to binomial distributions. On the first ball of each frame, each pin has equal probability p_1 of being knocked down and pins fall independently of each other. On the second ball of each frame, each remaining pin has equal probability p_2 of being knocked down, and again pins fall independently of each other. Spares and strikes are accounted for according to the regular rules of bowling. Under these conditions, we shall derive the average bowling score for a game with N pins, and find the correlation between the first roll and second roll in each frame. Moreover, we shall see that the second roll and the resulting sum of the two rolls still follow binomial distributions over the range of N pins.

The Binomial Distribution. A binomial distribution $X \sim b(n,p)$ counts the number of successes in n independent attempts, where p is the probability of success on each attempt. For $0 \le k \le n$, the probability of exactly k successes is given by

$$P\left(X=k\right)=\binom{n}{k}p^{k}\left(1-p\right)^{n-k},$$

the average number of successes is

$$E[X]=\sum_{k=0}^{n}kP\left(X=k\right)=np,$$