ALPHA-DISTANCE – A GENERALIZATION OF CHINESE CHECKER DISTANCE AND TAXICAB DISTANCE

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The Euclidean distance measures the shortest distance between two points. The taxicab distance [2] measures the distance between two points when only moves along axis-directions are permitted. If, in addition, the diagonal moves are also permitted, the distance between A and B is given by the Chinese checker distance [1]. Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two points in $\mathbb{R}^2$. Denote

$$\Delta_{AB} = \max\{|x_2 - x_1|, |y_2 - y_1|\} \quad \text{and} \quad \delta_{AB} = \min\{|x_2 - x_1|, |y_2 - y_1|\}.$$

The Euclidean distance between A and B is

$$d(A, B) = \sqrt{\Delta_{AB}^2 + \delta_{AB}^2}\ .$$

The taxicab distance between A and B is

$$d_T(A, B) = \Delta_{AB} + \delta_{AB}.$$

The Chinese checker distance between A and B is

$$d_C(A, B) = \Delta_{AB} + (\sqrt{2} - 1)\delta_{AB}.$$

In this paper we introduce a family of distances which include both Chinese checker distance and taxicab distance as special cases. For each $\alpha \in [0, \pi/4]$, we define the α-distance between A and B by

$$d_\alpha(A, B) = \Delta_{AB} + (\sec\alpha - \tan\alpha)\delta_{AB}.$$

Then, $d_0(A, B) = d_T(A, B)$ and $d_{\pi/4}(A, B) = d_C(A, B)$. Observe that if $\delta_{AB} > 0$, then

$$d_C(A, B) < d_\alpha(A, B) < d_T(A, B) \quad \text{for all} \quad \alpha \in (0, \pi/4).$$

When $\delta_{AB} = 0$, A and B lie on a horizontal or vertical line, and it follows that $d_C(A, B) = d_\alpha(A, B) = d_T(A, B) = d(A, B)$ for all $\alpha \in [0, \pi/4]$.

Clearly, $d_\alpha(A, B) = 0$ if and only if $A = B$, and $d_\alpha(A, B) = d_\alpha(B, A)$ for all $A, B \in \mathbb{R}^2$. The main result of this paper will be that

$$d_\alpha(A, B) \leq d_\alpha(A, C) + d_\alpha(C, B),$$