ALTERNATIVE PROOFS OF SOME RESULTS FROM ELEMENTARY ANALYSIS

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In an endeavor to encourage mathematics students to search for and study various methods of proof we have embarked on a program designed to develop and present approaches different from those found in the textbooks used by the students. As a result of this effort proofs of several well-known results from elementary analysis have been developed and presented. In this article we offer some of these proofs. Although we do not know if the proofs are new they are elegant, different from those usually found in books in analysis, and may be of use to students and teachers of the subject. First we present two proofs that a real-valued function which is 1-1 and continuous on an interval I in the reals is either increasing or decreasing on I. These proofs are different from and should be compared with that given in [9]. It is commented in [9] that it is possible to give a straightforward, but cumbersome, proof that involves keeping track of a lot of cases [4]. The proof given there dispenses with those unpleasant details, but is described by the author as "rather tricky." We then apply the so-called "creeping" method to establish three classical results: Dini's Lemma, the Bolzano-Weierstrass Theorem for the reals, and the Heine-Borel Theorem for Euclidean n-space. This method is abstracted and discussed in [6]. Proofs of Dini's Lemma and the Bolzano-Weierstrass Theorem are given in [6]. Those proofs should be compared with the proofs offered here. The proof of the Heine-Borel Theorem joins the method with induction on the dimension of the space and should be compared with that given in [8].

The article concludes with a nice induction proof of the Cauchy-Schwarz Inequality for complex numbers, a proof of the Lagrange Identity for complex numbers obtained by expressing the expansion of

$$\left|\sum_{k=1}^{n} \overline{z}_k w_k\right|^2$$

in a different form, and a generalization of the Lagrange Identity to inner-product spaces. These proofs should be compared with proofs found in books such as [1, 2, 5, 8], and with a proof in [3] using the upwards-downwards form of the principle of mathematical induction. We come to our results.

<u>Theorem 1</u>. A real-valued function f which is 1-1 and continuous on an interval I in the reals is either increasing or decreasing on I.