MODERN MATHEMATICAL MILESTONES: MORLEY'S MYSTERY

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A Euclidean result, discovered only in modern times, is the Morley Triangle Theorem. It reveals that the corresponding angle trisectors of any triangle intersect in the vertices of an equilateral triangle. This resulting triangle, named after Frank Morley (1860–1937), is to trisectors what the incenter of a triangle is to angle bisectors. Apparently overlooked by ancient geometers or hastily abandoned because of trisection and constructibility uncertainties, the problem came to light only a century ago. Though conjectured around 1900 by Frank Morley, resolution or rigorous proof was to await even more recent advancements. This beautiful and elegant Euclidean theorem, mysteriously unnoticed across the ages, thus belongs to the twentieth century.

One of the first to prove the Morley Triangle Theorem was M. T. Naraniengar in or around 1909. Today, proofs of many kinds are known; some are direct, some indirect. Both analytic and trigonometric proofs supplement the elephantine proofs of a purely geometric kind.

The problem continues to shed its abundant mysteries. Among these is the case for exterior angle trisectors as opposed to interior trisectors. The following figure, prepared for the writer by the university's graphics department, strongly suggests that corresponding exterior angle trisectors of any triangle also intersect in the vertices of an equilateral triangle. Personal proofs, first for special cases, and then in general by *MATHEMATICA,* rest on angle measure labels of the accompanying figure. An earlier proof is attributed to G. L. Niedhardt and V. Milenkovic in the late 1960s. The result was suggested in large measure by its bisector counterpart in the determination of the excenters of the general triangle.

Various questions, here unanswered, command our attention. Note that Z is the circumradius of the given triangle. Whereas side s (say DE) of the interior Morley triangle of triangle ABC is given by

$$s = 8Z \sin\frac{A}{3} \sin\frac{B}{3} \sin\frac{C}{3},$$