ON A SYMMETRIC FUNCTION OF THE PRIMITIVE ROOTS OF PRIMES

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1. Introduction. The elementary symmetric functions of n variables are:

$$s_1 = u_1 + u_2 + \cdots + u_n$$

$$s_2 = u_2u_1 + u_3u_1 + u_3u_2 + \cdots + u_nu_{n-1} = \sum_{i>j} u_iu_j$$

$$s_3 = \sum_{i>j>k} u_iu_ju_k$$

$$\vdots$$

$$s_n = \prod_{i=1}^{n} u_i.$$

In a previous paper [1] we investigated the elementary symmetric function s_1 of the primitive roots of a prime. The principal tool was the use of certain cyclotomic polynomials. The present work continues this line of investigation and considers the function s_2 of the primitive roots. Throughout, p denotes an odd prime, $d \geq 1$ is any divisor of $p-1$, and $\{g_i\}$ is the set of primitive roots of p.

2. Numerical Results. The first few odd primes, beginning with $p = 5$, yield the following simple results: $p = 5$: $s_2 \equiv 1 \pmod{p}$; $p = 7$: $s_2 \equiv 1 \pmod{p}$; $p = 11$: $s_2 \equiv 1 \pmod{p}$; $p = 13$: $s_2 \equiv -1 \pmod{p}$; $p = 17$: $s_2 \equiv 0 \pmod{p}$. The residues obtained strongly urge the computation of the residues modulo p of s_2 for many more primes. A sampling of these computations is given in Table 1. Since in [1] it was important to note whether $p-1$ is squarefree or not, Table 1 has been organized so that the primes p in columns (1)–(3) are those where $p-1$ is not squarefree; in columns (4) and (5) $p-1$ is squarefree.