THE POWER INTEGRAL AND THE GEOMETRIC SERIES

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The purpose of this note is to illustrate the importance of some ideas, methods, and techniques of the calculus classics in modern problems of teaching calculus. This approach, combined with the use of technology, provides, in our opinion, a positive contribution to retaining a solid theoretical foundation of the calculus and thus enhancing the success of the learning process. A number of such topics can be offered as student projects in the context of teaching Calculus, History of Mathematics, Differential Equations, Linear Algebra, Real Analysis, etc. These projects will also appeal to the interest of students in other areas of mathematics and at the same time develop their interest to a higher level.

The use of history in teaching mathematics will help convince the students that a college course in the history of mathematics should be primarily a mathematics course, and that a considerable amount of genuine mathematics should be injected in the subject. Such a course will be a study of the development of ideas that shape modern mathematical thinking and mathematicians who contributed those ideas. On the other hand, mathematics did not develop in a vacuum. It has always been an integral part of our life, thinking and culture. It has helped us to uncover the mysteries of nature and create technologies that not only change our world, but also our teaching methods. Therefore, in the teaching of calculus, it is very important to use many ideas and methods of the calculus classics which, in combination with modern technologies, will strengthen and broaden the students liberal education.

One of the examples that we suggest is the use of geometric progressions in the teaching of definite integrals (Fermat's idea).

Pierre Fermat (1601–65) is famous not only for his Last Theorem; he is also known as a founder of the modern theory of numbers and probability theory. He also did much to establish coordinate geometry and invented a number of methods for determining maxima and minima that were later of use to Newton in founding the calculus. Fermat recognized a principle in optics known as Fermat's Law. Fermat is also credited with a method of calculating areas under certain curves by partitioning the basic interval with a sequence of points whose coordinates form a geometric progression.

In this connection, we want to compare Fermat's method with the approach discussed in the note by Mathews [1] for the integral of the power function. In his capsule, Mathews found the integrals of $t^{1/2}$ and $t^{4/3}$ on $0 \leq t \leq x$ by Riemann sums with partition points

$$x_k = \frac{k^2 x}{n^2} \quad \text{and} \quad x_k = \frac{k^3 x}{n^3}, \quad k = 0, 1, \dots, n$$