A SIMPLE REDUCTION OF THE POINCARÉ DIFFERENTIAL EQUATION TO CAUCHY MATRIX FORM

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1. Introduction. G. D. Birkhoff was the first who tried to reduce each ordinary homogeneous linear differential equation with analytical coefficients to a certain canonical form [1, 2]. But F. R. Gantmacher [3] and P. Masani [4] gave counterexamples to Birkhoff's theorem [2]. Later, G. D. Birkhoff gave a true result [5] and noticed that his proof [6] is a special case of an important theorem given by D. Hilbert [7] and J. Plemelj [8]. But, H. L. Turrittin [9, 10] considered the reduction of more general systems of homogeneous linear differential equations up to Birkhoff's canonical form.

In the present paper, we prove the reduction of the Poincaré homogeneous differential equation of nth degree [11] with different regular singularities, up to the Cauchy matrix form [3].

2. A Simple Reduction. We will prove the following theorem.

Theorem. The Poincaré differential equation

$$P_n(x)y^{(n)} = \sum_{i=0}^{n-1} P_i(x)y^{(i)}, \tag{1}$$

where $P_n(x) = \prod_{i=1}^{n}(x-d_i) = \varphi_n(x)$, $(d_i \neq d_j;\ i \neq j)$ reduces to the Cauchy matrix form

$$(xI - D)\frac{dY}{dx} = QY, \tag{2}$$

where

$$D = \mathrm{diag}\ (d_1, d_2, \ldots, d_n), \tag{3}$$

$$Q = \begin{bmatrix} q_{1,1} & 1 & 0 & \cdots & 0 & 0 \\ q_{2,1} & q_{2,2} & 1 & \cdots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ q_{n-1,1} & q_{n-1,2} & q_{n-1,3} & \cdots & q_{n-1,n-1} & 1 \\ q_{n,1} & q_{n,2} & q_{n,3} & \cdots & q_{n,n-1} & q_{n,n} \end{bmatrix}, \tag{4}$$