# CONSTRUCTING PYTHAGOREAN TRIPLE PRESERVING MATRICES 

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1. Introduction. In the first paper we found that a $3 \times 3$ matrix which is a PTPM, i.e., converts a Pythagorean triple into a Pythagorean triple, has to be of the type H , where

$$
H=\left(\begin{array}{ccc}
\left(\left(r^{2}-t^{2}\right)-\left(s^{2}-u^{2}\right)\right) / 2 & r s-t u & \left(\left(r^{2}-t^{2}\right)+\left(s^{2}-u^{2}\right)\right) / 2 \\
r t-s u & r u+s t & r t+s u \\
\left(\left(r^{2}+t^{2}\right)-\left(s^{2}+u^{2}\right)\right) / 2 & r s+t u & \left(\left(r^{2}+t^{2}\right)+\left(s^{2}+u^{2}\right)\right) / 2
\end{array}\right) .
$$

We also recall that every Pythagorean triple has the form $\left(m^{2}-n^{2}, 2 m n, m^{2}+\right.$ $n^{2}$ ) and furthermore, is a PPT if the pair $(m, n)$ satisfies the four conditions listed below.

I-1. $m, n$ are positive integers
I-2. $m>n$
I-3. $\operatorname{gcd}(m, n)=1$
I-4. $m+n \equiv 1(\bmod 2)$.
We have seen that $\left(m^{2}-n^{2}, 2 m n, m^{2}+n^{2}\right) H=\left(M^{2}-N^{2}, 2 M N, M^{2}+N^{2}\right)$, where the pairs $(m, n)$ and $(M, N)$ are related by the matrix equation

$$
(m, n)\left(\begin{array}{cc}
r & s \\
t & u
\end{array}\right)=(M, N)
$$

If we start with a pair ( $m, n$ ) which satisfies I-1 to I-4 and multiply it with the matrix

$$
R=\left(\begin{array}{ll}
r & s \\
t & u
\end{array}\right)
$$

the new pair $(M, N)$ may not satisfy I-1 to I-4. To assure that $(M, N)$ also satisfies I- 1 to I-4, suitable restrictions must be imposed on $r, s, t$, and $u$. These are listed as $\mathrm{R}-1$ to $\mathrm{R}-4$ below.

