## SOLUTIONS

No problem is ever permanently closed. Any comments, new solutions, or new insights on old problems are always welcomed by the problem editor.

**71**. [1994, 98; 1995, 91] *Proposed by Ronnie Gupton, Larry Hoehn, and Jim Ridenhour, Austin Peay State University, Clarksville, Tennessee.*

Provide a non-calculus solution to the following problem on page 530 of James Stewart's *Calculus* (2nd ed.), Brooks/Cole Publishing Company, 1991.

"A cow is tied to a silo with radius $r$ by a rope just long enough to reach the opposite side of the silo. Find the area available for grazing by the cow."

*Comment by Lamarr Widmer, Messiah College, Grantham, Pennsylvania.*

It seems to me that a solution that uses a limit does not really qualify as a "non-calculus" solution. Nonetheless, it is a very interesting solution and a nice alternative to the usual solution to this cow grazing problem.

**75**. [1994, 160; 1995, 146] *Proposed by Leonard L. Palmer, Southeast Missouri State University, Cape Girardeau, Missouri.*

"Prove that $n(n+1)$ is never a square for $n > 0$" is a problem in *Elementary Number Theory* by Underwood Dudley. Generalize this problem by showing that $n(n+1) \neq t^k$ for $t$ an integer and $k \geq 2$.

*Comment by Kandasamy Muthuvel, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.*

Problem 75 may be generalized in the following way.

Generalization. Suppose $x$ and $y$ are relatively prime and $k$ is a positive integer such that $k > 1$ and $0 < |x-y| \leq k$. Then $xy \neq t^k$ for any integer $t$.

Proof. Suppose $xy = t^k$ for some integer $t$. Since $x$ and $y$ are relatively prime, $x = a^k$ and $y = b^k$ for some positive integers $a$ and $b$. Then

$$x - y = a^k - b^k = (a-b)(a^{k-1} + a^{k-2}b + \cdots + b^{k-1}).$$

Note that $a, b \geq 1$ and since $|x-y| > 0$, $a$ or $b$ is greater than 1. Hence

$$|x-y| > |a-b|k \geq k.$$

This contradicts the fact that $|x-y| \leq k$ and the result follows.